Formal Appendix

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Abstract

This note provides formal derivations of claims made in "The Political Economy of Natural Resource Funds" (Chapter 8 of *Escaping the Resource Curse*, New York: Columbia University Press). We provide a two period model in in which two individuals take turns in dividing a non-renewable asset. The problem is analogous to that facing countries endowed with a natural resource or other fund. Insecurity over control of the division combined with policy disagreement yield faster than optimal consumption.

1 Model

We assume there is a "fund" of accumulated past natural resource revenues of size $X = 1$. In each of two periods a non-negative amount $x_{i,t}$ can be allocated to projects benefiting each of two groups $i \in \{a, b\}$, subject to $\sum_i \sum_t x_{i,t} \leq 1$, $t \in \{1, 2\}$.

There are two potential policy makers, one associated with each group, also denoted by a and b. They have divergent preferences over the welfare of two different constituencies.

The instantaneous utility function of policy maker i is given by

$$v_{i,t} = \alpha x_{i,t}^{\rho} + (1-\alpha) x_{j,t}^{\rho}, i \neq j, \rho \in (0,1), \alpha > .5.$$

In addition, policy makers maximize over present utility and all expected discounted future utility, captured by discount factor $\delta \in (0, 1)$. Hence expected utility in period 1 is given by $v_i = v_{i,1} + \delta v_{i,2}$ while in period 2 it is simply $v_{i,2}$.

2 Equilibrium with no power rivalry

We begin by examining the optimal decision making in cases in which the security of tenure of the period 1 policy maker is guaranteed.

Claim 1 *In the absence of competition, the policy maker gives a fixed share to each group in each period. The policy maker's own group gets the largest share, and the share increases the more biased the policy maker is. Moreover, the more slowly marginal utility diminishes, the more unequal is the allocation.*

Proof. Denote the aggregate spending in period t by $X_t = x_{a,t} + x_{b,t}$. With $X_2 = 1 - X_1$, policy-maker i in power at time 2 maximizes:

$$v_{i,2} = \alpha x_{i,2}^{\rho} + (1-\alpha) x_{j,2}^{\rho} \text{ subject to } x_{i,2} + x_{j,2} \leq X_2$$

The first order condition for this problem is given by:

$$\alpha \rho x_{i,2}^{\rho-1} - \rho(1-\alpha)(X_2 - x_{i,2})^{\rho-1} = 0 \qquad (1)$$

which is solved by:

$$x_{i,2} = \frac{\alpha^{\frac{1}{1-\rho}}}{(1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}}} X_2 = \lambda(\alpha) X_2 \tag{2}$$

where the function $\lambda(z)$ is given by

$$\lambda(z) \equiv \frac{z^{\frac{1}{1-\rho}}}{(1-z)^{\frac{1}{1-\rho}} + z^{\frac{1}{1-\rho}}} \tag{3}$$

Note that second period utility equals $[\alpha^{\frac{1}{1-\rho}} + (1-\alpha)^{\frac{1}{1-\rho}}]^{1-\rho} X_2^\rho$. Hence, conditional on this solution in the second period, the first period maximization problem is:

$$\max_{\{x_{k,1}\}: x_{i,1}+x_{j,1} \leq 1} U = \alpha x_{i,1}^\rho + (1-\alpha) x_{j,1}^\rho + \delta [\alpha^{\frac{1}{1-\rho}} + (1-\alpha)^{\frac{1}{1-\rho}}]^{1-\rho} (1 - x_{i,1} - x_{j,1})^\rho \tag{4}$$

The first order conditions for this problem are:

$$\begin{aligned} \rho\alpha x_{i,1}^{\rho-1} - \delta\rho[\alpha^{\frac{1}{1-\rho}} + (1-\alpha)^{\frac{1}{1-\rho}}]^{1-\rho}[1 - x_{i,1} - x_{j,1}]^{\rho-1} &= 0 \quad (5) \\ \rho(1-\alpha) x_{j,1}^{\rho-1} - \delta\rho[\alpha^{\frac{1}{1-\rho}} + (1-\alpha)^{\frac{1}{1-\rho}}]^{1-\rho}[1 - x_{i,1} - x_{j,1}]^{\rho-1} &= 0 \end{aligned}$$

yielding the solution:

$$\begin{aligned} x_{i,1} &= \frac{\alpha^{\frac{1}{1-\rho}}}{\left((1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}}\right)\left(1 + \delta^{\frac{1}{1-\rho}}\right)} \quad (6) \\ x_{j,1} &= \frac{(1-\alpha)^{\frac{1}{1-\rho}}}{\left((1-\alpha)^{\frac{1}{1-\rho}} + \alpha^{\frac{1}{1-\rho}}\right)\left(1 + \delta^{\frac{1}{1-\rho}}\right)} \end{aligned}$$

Note furthermore that with this solution we have: $X_1 = x_{i,1} + x_{j,1} = \frac{1}{1+\delta^{\frac{1}{1-\rho}}}$ and hence $x_{i,1}$, can then be written more simply as:

$$x_{i,1} = \lambda(\alpha) X_1 \tag{7}$$

We therefore have that the same share of the allocated pie is allocated to each group in each time period. Indeed it is easy to check that given any intertemporal allocation the between-group allocation is determined by $\lambda(\alpha)$.

The final parts of the claim can be checked by observing that:

1. $z > .5 \to \lambda(z) > .5$,

2. $\frac{\partial \lambda(z)}{\partial z} > 0$ and

3. $\frac{\partial \lambda(z)}{\partial \rho} \geq 0 \leftrightarrow z \geq .5$

■

Claim 1 establishes Findings 1 and 2 from the text. Note that efficiency and optimality for the incumbent follows from the fact that she decides the entire allocation across time periods and across groups, without constraints.

3 Equilibrium with power rivalry and exogenous transition probabilities

We now consider the case where the policy maker's tenure is uncertain. We analyze the policy chosen by i in period 1 when she confronts a probability q of being removed from office and replaced by j at the end of period 1.

Claim 2 (Competition) *Period 1 expenditure is increasing in q.*

Proof. We use Equation 2 and drop the argument for $\lambda(\alpha)$, writing $\lambda = \lambda(\alpha)$. The incumbent's period 2 payoff if she remains in office is given by:

$$v_{a,2}|\text{survives} = \psi_1 X_2^\rho$$

where $\psi_1 \equiv \alpha\lambda^\rho + (1-\alpha)(1-\lambda)^\rho$.

If she loses office she will get:

$$v_{a,2}|\text{removed} = \psi_2 X_2^\rho$$

where $\psi_2 \equiv \alpha(1-\lambda)^\rho + (1-\alpha)\lambda^\rho$.

Note that ψ_1 reflects the benefits to player 1 from controlling the division of X_2 and ψ_2 reflects the benefits to player 1 from the other player controlling the division of X_2. Clearly $\psi_1 > \psi_2$. In addition, the ratio $\psi \equiv \frac{\psi_2}{\psi_1} \in [0,1)$ is the incumbent's marginal rate of substitution between spending according to the challenger's preferences and according to her own preferences. It serves as a measure of policy agreement and approaches its upper bound of 1 when policy disagreement disappears (as α approaches .5).

The incumbent's expected period 2 benefit is therefore:

$$[q\psi_2 + (1-q)\psi_1]\, {X_2}^{\rho} \tag{8}$$

Define:

$$\begin{aligned} \gamma &\equiv \delta\,[q\psi_2 + (1-q)\psi_1] && (9)\\ &= \delta\,[\psi_1 - q(\psi_1 - \psi_2)] && (10) \end{aligned}$$

Then using Equations 7 and 8, and substituting $1 - X_1$ for X_2, her Period 1 maximization problem can be written as a function of X_1:

$$\max_{X_1 \in [0,1]} \psi_1 {X_1}^{\rho} + \gamma(1 - X_1)^{\rho} \tag{11}$$

The first order condition is:

$$\rho\psi_1 X_1^{\rho-1} - \gamma\rho(1 - X_1)^{\rho-1} = 0 \tag{12}$$

Solving for an equilibrium value of X_1^* yields:

$$\begin{aligned} X_1^* &= \frac{1}{\frac{\gamma}{\psi_1}^{\frac{1}{1-\rho}} + 1}\\ &= \frac{1}{1 + \delta^{\frac{1}{1-\rho}}\,(q\psi + (1-q))^{\frac{1}{1-\rho}}} \end{aligned}$$

With $\psi \in (0,1)$ it is easy to check that $\frac{\partial X_1^*}{\partial q} > 0$, which proves the claim. ■

Claim 2 supports Finding 3 in the text. In addition, to establish Finding 4, observe that $\frac{\partial X_1^*}{\partial \psi} < 0$ and $\frac{\partial^2 X_1^*}{\partial \psi \partial q} < 0$. The latter of these two inequalities can be checked by noting that $sgn\left[\frac{\partial^2 X_1^*}{\partial \psi \partial q}\right] = -sgn[\frac{\partial^2}{\partial \psi \partial q}(q\psi + (1-q))] < 0$.

This distortion of the incumbent's spending choice (relative to the efficient time path of spending which she chooses when she is certain to remain in power) induces an *ex ante* re-distribution towards the incumbent's group relative to the optimum. This is Finding 5 in the text. For a given amount of period 1 spending, X_1, the expected (undiscounted) share of the fund allocated to the incumbent's group, i, is

$$\begin{aligned}\mathsf{E}(x_{i,1}+x_{i,2}) &= \lambda X_1 + q(1-\lambda)(1-X_1) + (1-q)\lambda[1-X_1] \\ &= \lambda - [2\lambda-1]\,q + [2\lambda-1]\,qX_1\end{aligned} \tag{13}$$

Hence:

$$\frac{\partial \mathsf{E}(x_{i,1}+x_{i,2})}{\partial X_1} = [2\lambda-1]\,q$$

Since $\lambda > .5$ we have $\frac{\partial \mathsf{E}(x_{i,1}+x_{i,2})}{\partial X_1} > 0$, and so $\frac{\partial \mathsf{E}(x_{j,1}+x_{j,2})}{\partial X_1} < 0$. So an increase in X_1, *ceteris paribus*, increases the allocation to the incumbent's group. This is not surprising; it simply reflects the fact that when the incumbent shifts spending from the future to the present, she spends more according to her preferred allocation, and less in a period when there is a nonzero probability the challenger will get to choose a different allocation.

At the same time, this does *not* mean that the increase in q that leads the incumbent to raise X_1 results in an increase in the allocation to the incumbent's group. To answer that question we need to take account of the direct and the indirect effects of the rise in q. Equation 13 shows that $\mathsf{E}(x_{i,1}+x_{i,2}) = \lambda - q\,[2\lambda-1]\,[1-X_1]$, which can be interpreted as share λ, less some positive amount that is itself a function of q, λ and the equilibrium X_2. Hence immediately we see that for any $q > 0$, total expected gains to group a are less than they would have been for $q = 0$. We next consider whether the returns to group i are also strictly declining in q.

We have:

$$\frac{\partial \mathsf{E}(x_{i,1}+x_{i,2})}{\partial q} = [2\lambda-1]\left[q\frac{\partial X_1}{\partial q} - (1-X_1)\right]$$

Hence:

$$\frac{\partial \mathsf{E}(x_{i,1}+x_{i,2})}{\partial q} > 0 \leftrightarrow q\frac{\partial X_1}{\partial q} > 1 - X_1$$

Since in equilibrium we have $X_1^* = \frac{1}{1+\delta^{\frac{1}{1-\rho}}(q\psi+(1-q))^{\frac{1}{1-\rho}}}$ we have:

$$\frac{\partial X_1^*}{\partial q} = \frac{(q\psi+(1-q))^{\frac{\rho}{1-\rho}}\,\frac{1-\psi}{1-\rho}\delta^{\frac{1}{1-\rho}}}{\left[1+\delta^{\frac{1}{1-\rho}}\,(q\psi+(1-q))^{\frac{1}{1-\rho}}\right]^2}$$

Putting these pieces together we have:

$$
\begin{aligned}
q\frac{\partial X_1^*}{\partial q} &> 1 - X_1^* \\
&\leftrightarrow \\
q\frac{(q\psi+(1-q))^{\frac{\rho}{1-\rho}}\frac{1-\psi}{1-\rho}\delta^{\frac{1}{1-\rho}}}{\left[1+\delta^{\frac{1}{1-\rho}}(q\psi+(1-q))^{\frac{1}{1-\rho}}\right]^2} &> \frac{\delta^{\frac{1}{1-\rho}}(q\psi+(1-q))^{\frac{1}{1-\rho}}}{1+\delta^{\frac{1}{1-\rho}}(q\psi+(1-q))^{\frac{1}{1-\rho}}} \\
&\leftrightarrow \\
\frac{q\frac{1-\psi}{1-\rho}-q\psi-(1-q)}{[q\psi+(1-q)]^{\frac{2-\rho}{1-\rho}}} &> \delta^{\frac{1}{1-\rho}}
\end{aligned}
$$

This condition can be satisfied for combinations in which δ and ψ are low relative to q and ρ (it is, for example, satisfied for $\delta = \psi = .25$ and $q = \rho = .5$).

Hence if leaders are impatient and the challenger is strong then *the stronger is the challenger the worse off is his constituency.* The intuition behind this result is the following. It is beneficial for the challenger to go from a zero chance of gaining power to a small positive chance. But as the chance of gaining power increases, the incumbent protects herself against a possible loss of power by shifting spending away from the future. Under the conditions described in the previous paragraph, if the probability of a power change is high, a further increase in the challenger's prospects of gaining power induces the incumbent to shift spending so rapidly to the present (as she becomes increasingly unlikely to control the allocation in the future) that the challenger ends up worse off.

In all events, the expenditure profile under uncertainty is inefficient. This is Finding 6 in the text and is established by the next claim.

Claim 3 (Inefficiency) *For any $q > 0$ there exist alternative expenditure profiles that are preferred ex ante by both policy makers, in which the challenger, if he takes power, would allocate a higher share of Period 2 expenditure to the incumbent's group than in the equilibrium expenditure profile.*

Proof. Assume that b can precommit to spend share λ_b on group a in the event that she comes to power. In particular consider a small change in the allocation made by b from the equilibrium allocation $1 - \lambda$. Clearly a benefits from such a change, since she can make the same choices as before, but does strictly better in the event that b comes to power. In fact, however, since the marginal expected utility of period 2 expenditures increases, she will modify her own period 1 expenditure somewhat.

The incumbent's Period 2 expected utility is now given by:

$$q\psi_2' + (1-q)\psi_1 X_2^{\rho} \tag{14}$$

where

$$\psi_2'(\lambda_b) \equiv \alpha(\lambda_b)^{\rho} + (1-\alpha)(1-\lambda_b)^{\rho}$$

Her Period 1 first order condition is now given by:

$$\psi_1 X_1^{\rho-1} - \delta\left(q\psi_2'(\lambda_b) + (1-q)\psi_1\right)(1-X_1)^{\rho-1} = 0$$

The induced change in first period expenditure is given by applying the implicit function theorem to the first-order condition for the incumbent's maximization problem (Equation 11):

$$\begin{aligned}
\frac{dX_1}{d\lambda_b} &= -\frac{\frac{\partial\left[\frac{\partial Ev_1}{\partial X_1}\right]}{\partial \lambda_b}}{\frac{\partial \frac{\partial Ev_1}{\partial X_1}}{\partial X_1}} && (15)\\
&= \frac{(1-X_1)^{\rho-1}\delta q \frac{\partial \psi_2'(\lambda_b)}{\partial \lambda_b}}{\frac{d^2 Ev_1}{dX_1^2}} && (16)\\
&= \frac{(1-X_1)^{\rho-1}\delta q\rho\left[\alpha\lambda_b^{\rho-1} - (1-\alpha)(1-\lambda_b)^{\rho-1}\right]}{\frac{d^2 Ev_1}{dX_1^2}}\\
&< 0
\end{aligned}$$

where the final signing comes from the facts that (i) the second order condition for the maximization problem is satisfied ($\frac{d^2 Ev_1}{dX_1^2} < 0$); and (ii) evaluated at $\lambda_b = 1-\lambda$, we have $\lambda_b^{\rho-1} > (1-\lambda_b)^{\rho-1}$ and $\alpha > (1-\alpha)$ and so $\alpha\lambda_b^{\rho-1} - (1-\alpha)(1-\lambda_b)^{\rho-1} > 0$.

This change in Period 1 expenditure is sufficient to produce an improvement for b that more than offsets the loss in utility from the small increase in Period 2 allocation to group a. To see this, note that the change in b's utility from the increase in λ_b can be written:

$$
\begin{aligned}
\frac{dEv_b}{d\lambda_b} &= \frac{\partial Ev_b}{\partial \lambda_b} + \frac{\partial Ev_b}{\partial X_1}\frac{dX_1}{d\lambda_b} + \frac{\partial Ev_b}{\partial X_2}\frac{dX_2}{dX_1}\frac{dX_1}{d\lambda_b} \\
&= \underbrace{\frac{\partial Ev_b}{\partial \lambda_b}}_{0} + \underbrace{\left[\frac{\partial Ev_b}{\partial X_1} - \frac{\partial Ev_b}{\partial X_2}\right]}_{-} \underbrace{\frac{dX_1}{d\lambda_b}}_{-} \\
&> 0
\end{aligned}
$$

We have $\frac{\partial Ev_b}{\partial \lambda_b}$ from the envelope theorem; $\left[\frac{\partial Ev_b}{\partial X_1} - \frac{\partial Ev_b}{\partial X_2}\right] < 0$, since at the equilibrium Period 1 expenditure as determined by the incumbent, the marginal utility to of expenditure in Period 2 is higher than that in Period 1 for a challenger with different priorities (and equal if there is no policy disagreement)[1]; and $\frac{dX_1}{d\lambda_b} < 0$ from Equation 15. ■

4 Endogenous q

We now consider the case in which q is a function of total expenditure in period 1. In this case we analyze the expenditure and transition probability

[1]To see this formally:

Evaluated in equilibrium with $\lambda_b = 1 - \lambda$ we have:

$$Ev_b = \psi_2 X_1^{*\rho} + \delta\left(q\psi_1 X_2^{*\rho} + (1-q)\psi_2 X_2^{*\rho}\right)$$

The statement $\left[\frac{\partial Ev_b}{\partial X_1} - \frac{\partial Ev_b}{\partial X_2}\right] < 0$ is equivalent to the statements:

$$
\begin{aligned}
\rho\psi_2 X_1^{*\rho-1} &< \delta\left(q\rho\psi_1 X_2^{*\rho-1} + (1-q)\rho\psi_2 X_2^{*\rho-1}\right) \\
&\leftrightarrow \\
\frac{\psi}{\delta}\left(\frac{X_1^*}{1-X_1^*}\right)^{\rho-1} &< q + (1-q)\psi \\
&\leftrightarrow \\
\frac{\psi}{\delta}\left(\frac{1}{\delta^{\frac{1}{1-\rho}}\left(q\psi + (1-q)\right)^{\frac{1}{1-\rho}}}\right)^{\rho-1} &< q + (1-q)\psi \\
&\leftrightarrow \\
\psi\left(q\psi + (1-q)\right) &< q + (1-q)\psi \\
&\leftrightarrow \\
\psi\psi &< 1
\end{aligned}
$$

that arises in equilibrium. Focusing on the equilibrium q we ask how would expenditure change if instead of being exogenous, this transition probability were instead endogenous to expenditure. Our enquiry establishes the following claim:

Claim 4 *Let the probability that the incumbent is turned out of office be given by a function $q(X_1)$ with the property that the maximand $v_a(X_1) = \psi_1 X_1^\rho + \delta\left[\psi_1 - q(X_1)(\psi_1 - \psi_2)\right](1 - X_1)^\rho$ is concave in X_1. Let X_1^* denote equilibrium expenditure in this case and let $q^* = q(X_1^*)$ denote the corresponding probability of losing office in equilibrium. Consider the comparison case in which the probability of losing office is exogenously given by $q' = q^*$ and the corresponding equilibrium expenditure is given by X_1'. If $\frac{\partial q}{\partial X_1} > 0$ (< 0) then $X_1' < (>) X_1^*$. Furthermore, the greater is $|\frac{\partial q}{\partial X_1}|$ the greater is $|X_1' - X_1^*|$.*

Proof. Following Equation 9 define:

$$\gamma(X_1) = \delta\left[\psi_1 - q(X_1)(\psi_1 - \psi_2)\right] \tag{17}$$

and note that since $\psi_1 - \psi_2 > 0$, $sgn\left[\frac{\partial\gamma}{\partial q}\frac{\partial q}{\partial X_1}\right] = -sgn\left[\frac{\partial q}{\partial X_1}\right]$.

As in Equation 11, the Period 1 maximization problem can be written as a function of X_1 alone:

$$\max_{X_1 \in [0,1]} \psi_1 X_1^\rho + \gamma(X_1)(1 - X_1)^\rho \tag{18}$$

The first order condition is:

$$\rho\psi_1 X_1^{\rho-1} - \rho\gamma(X_1)(1 - X_1)^{\rho-1} + \frac{\partial\gamma}{\partial q}\frac{\partial q}{\partial X_1}(1 - X_1)^\rho = 0 \tag{19}$$

Now, let X_1^* denote equilibrium expenditure in this case and let $q^* = q(X_1^*)$ denote the corresponding probability of losing office in equilibrium. Consider the comparison case in which the probability of losing office is exogenously given by $q' = q^*$ and the corresponding equilibrium expenditure is given by X_1'.

Letting $\mathsf{E}v_a(X_1|q')$ denote expected utility in this comparison case, the first order condition for the endogenous survival case (Equation 19) can be written:

$$\frac{\partial \mathsf{E}v_a(X_1|q')}{\partial X_1} = -\frac{\partial\gamma}{\partial q}\frac{\partial q}{\partial X_1}(1 - X_1)^\rho$$

Recall that $sgn\left[\frac{\partial\gamma}{\partial q}\frac{\partial q}{\partial X_1}\right] = -sgn\left[\frac{\partial q}{\partial X_1}\right]$ and hence $sgn\left[\frac{\partial \mathsf{E}v_a(X_1|q')}{\partial X_1}\right] = sgn\left[\frac{\partial q}{\partial X_1}\right]$.

With $\frac{\partial q}{\partial X_1}$ negative (that is, survival is more likely with more first period expenditure), $\frac{\partial \mathsf{E}v_a(X_1|q')}{\partial X_1}$ is negative but since $\mathsf{E}v_a(X_1|q')$ is concave, it must be the case that X_1 in the endogenous case is set at a higher level than in the exogenous case (for which $\frac{\partial \mathsf{E}v_a(X_1|q')}{\partial X_1} = 0$). Likewise the more negative is $\frac{\partial q}{\partial X_1}$, the larger is X_1. A similar argument holds for $\frac{\partial q}{\partial X_1}$ positive. Together these imply (i) if $\frac{\partial q}{\partial X_1} > 0$ (< 0) then $X_1' < (>) X_1^*$ and (ii) the greater is $|\frac{\partial q}{\partial X_1}|$ the greater is $|X_1' - X_1^*|$. ■

5 Transparency

We close by explicitly modelling the population's choice of whether to support the incumbent and examining how their behavior, and consequently that of the incumbent policy maker, are affected by the degree of transparency of the incumbent's actions. We present a Bayesian model in which voters are fully rational. To focus on the time path of aggregate spending, we leave interest group rivalry behind and assume that voters have common goals. Uncertainty is centered around the type of the incumbent. We assume that there are two types of policy makers: A patient type with discount rate δ^H that is equal to the discount rate of voters (the high type thus "represents" the voters in the sense of having similar intertemporal preferences); and an impatient type with discount rate $\delta^L < \delta^H$. We let $r \in (0, 1)$ denote the share of patient potential policy makers in the population of potential policy makers.

In the last period, it does not matter what type of policy maker is in power, since there is no choice to make (policy makers simply spend whatever is left of the fund). Thus rational voters have no reason to support or oppose the incumbent just before the last period. To model the probability of a power change as a result of rational voter behavior, we therefore assume three periods. Disagreement between policy makers and between policy makers and voters centers on the time allocation of expenditures, and voters will act on their preferences in an election between Periods 1 and 2 to try to secure their preferred trade-off between expenditure in Periods 2 and 3. We assume that preferences are given by:

$$U_i(X_1, X_2) = u(X_1) + \delta^i u(X_2) + \left(\delta^i\right)^2 u(1 - X_1 - X_2)$$

After observing the Period 1 actions of a given policy maker, the population has the choice to retain the policy maker or remove him from office. They have only imperfect knowledge of the incumbent's actions, however: There is only a probability p that they will see them. Thus p is a measure of fund transparency. If they remove the policy maker they can replace him only with a random policy maker drawn from the population of all policy makers.

As already noted, policy makers in Period 3 have no discretion and hence decisions taken in Period 2 fully determine expenditure in both Period 2 and Period 3. Given this, let $X_1^*(\delta^i)$ denote the optimal choice of Period 1 expenditure for an incumbent of type δ^i who is assured of remaining in office in Period 2 and let $X_2^*(\delta^i|X_1)$ denote the optimal choice of Period 2 expenditure for an incumbent of type δ^i conditional on X_1 having been spent in period 1.

Let V_i^* denote the maximum value that can be gained by an incumbent of type i guaranteed to be in power in all periods. By definition,

$$V_i^* = u\left(X_1^*(\delta^i)\right) + \delta^i u\left(X_2^*(\delta^i|X_1^*(\delta^i))\right) + \left(\delta^i\right)^2 u\left(1 - X_1^*(\delta^i) - X_2^*(\delta^i|X_1^*(\delta^i))\right)$$

Let $V_{i,j}(X_1)$ denote the value in Period 2 for a Type i policy maker if a type j decision maker is in power in Period 2 conditional upon X_1 having been spent in period 1. Since we assume that all policy makers optimize according to their own preferences, we have

$$V_{i,j}(X_1) = u\left(X_2^*(\delta^j|X_1)\right) + \delta^i u\left(1 - X_1 - X_2^*(\delta^j|X_1)\right)$$

We let μ denote the voter's belief at the beginning of Period 2 that the Period 1 incumbent is a patient type. We assume that the Period 1 policy maker is drawn from the pool of possible policy makers. The prior belief that the Period 1 incumbent is patient is thus given by $\tilde{\mu} = r$.

Under these conditions we establish the following claim.

Claim 5 *If transparency is sufficiently high, incumbent policy makers who are impatient types mimic the patient types by choosing the latter's preferred policy in Period 1. More precisely: The following strategies and beliefs constitute a weak perfect Bayesian equilibrium if and only if* $p \geq \frac{1}{r}\frac{V_L^* - [u(X_1^*(\delta^H)) + \delta^L V_{L,L}(X_1^*(\delta^H))]}{\delta^L\left[V_{L,L}(X_1^*(\delta^L)) - V_{L,H}(X_1^*(\delta^L))\right]}$.

- *H types implements* $X_1^*(\delta^H)$ *if in office in period 1,* $X_2^*(\delta^H|X_1)$ *if in office in period 2 and* $1 - X_1 - X_2$ *if in office in Period 3.*

- *L types implement $X_1^*(\delta^H)$ if in office in period 1, $X_2^*(\delta^L|X_1)$ if in office in period 2 and $1 - X_1 - X_2$ if in office in Period 3.*

- *Voters' first-period beliefs are given by: $\tilde{\mu} = r$. In Period 2, if they do not observe the policy maker's actions, their beliefs are $\mu = \tilde{\mu}$. If, in contrast, they do observe the policy maker's actions, their beliefs are $\mu = 0$ if $X_1 > X_t^*(\delta^H)$, $\mu = r$ if $X_1 = X_t^*(\delta^H)$ and $\mu = 1$ if $X_1 < X_t^*(\delta^H)$.*

- *Voters replace the incumbent with a random member of possible policy makers if and only if $\mu < r$.*

Proof. Since in the proposed equilibrium, Period 1 incumbents pool (low types mimic high types), voters second-period beliefs are the same as their prior beliefs in equilibrium. For out of equilibrium play by incumbents we need place no constraint on voter beliefs. We note however that the beliefs posited in the equilibrium are consistent with a reasonable voter supposition that overspending signals an impatient type and underspending a patient type.

Given voter strategies, H types clearly have an incentive to choose their preferred strategies, knowing that they will be re-elected to implement them.

Given the actions of the other players, an impatient incumbent will spend prudently in Period 1 if and only if:

$$\begin{aligned} u(X_1^H) + \delta^L V_{L,L}(X_1^H) &\geq u(X_1^L) + pr\delta^L V_{L,H}(X_1^L) + (1-pr)\delta^L V_{L,L}(X_1^L) \\ &= u(X_1^L) + \delta^L V_{L,L}(X_1^L) - pr\delta^L \left[V_{L,L}(X_1^L) - V_{L,H}(X_1^L)\right] \\ &= V_L^* - pr\delta^L \left[V_{L,L}(X_1^L) - V_{L,H}(X_1^L)\right] \end{aligned}$$

where $X_1^H \equiv X_1^*(\delta^H)$ and $X_1^L \equiv X_1^*(\delta^L)$.

From optimality we have $V_{L,L}(X) > V_{L,H}(X)$ and $V_{H,H}(X) > V_{H,L}(X)$ for all $X \in (0,1]$ and $u(X_1^H)+\delta^L V_{L,L}(X_1^H) < u(X_1^L)+\delta^L V_{L,L}(X_1^L)$. Thus the condition for an impatient policy maker to spend prudently can be written:

$$p \geq \frac{1}{r}\frac{V_L^* - [u(X_1^H) + \delta^L V_{L,L}(X_1^H)]}{\delta^L\,[V_{L,L}(X_1^L) - V_{L,H}(X_1^L)]} > 0$$

which is exactly the condition of the claim. ∎

Under the same conditions, we establish the converse claim, that impatient incumbents may overspend in Period 1 when transparency is too low.

Claim 6 *If transparency is sufficiently low, patient and impatient policy makers adopt different strategies in every period, and in particular, impatient incumbents spend more in Period 1 than do patient incumbents. Specifically, the following strategies and beliefs constitute a weak perfect Bayesian equilibrium if and only if* $p \leq \frac{1}{r}\frac{V_L^* - [u(X_1^*(\delta^H)) + \delta^L V_{L,L}(X_1^*(\delta^H))]}{\delta^L \left[V_{L,L}(X_1^*(\delta^L)) - V_{L,H}(X_1^*(\delta^L))\right]}$.

- *H types implements $X_1^*(\delta^H)$ in period 1, $X_2^*(\delta^H|X_1)$ in period 2 and $1 - X_1 - X_2$ in Period 3.*
- *L types implement $X_1^*(\delta^L)$ in period 1, $X_2^*(\delta^L|X_1)$ in period 2 and $1 - X_1 - X_2$ in Period 3.*
- *Voters' first-period beliefs are given by: $\tilde{\mu} = r$. In Period 2, beliefs are are $\mu = \tilde{\mu}$ if they do not observe the policy maker's actions. If they do observe it, $\mu = 0$ if $X_1 > X_t^*(\delta^H)$ and $\mu = 1$ if $X_1 \leq X_t^*(\delta^H)$.*
- *Voters replace the incumbent with a random member of possible policy makers if and only if $\mu < r$.*

Proof. With no new information, voters second-period beliefs are the same as their prior beliefs in equilibrium. If policy makers follow equilibrium strategies then voter beliefs are consistent with Bayes' rule. As before for out of equilibrium play by incumbents we need place no constraint on voter beliefs. We note however that the beliefs posited in the equilibrium are consistent with a reasonable voter supposition that overspending signals an impatient type and underspending a patient type.[2] Thus the posited beliefs are consistent with the conditions for an equilibrium.

To prove that the proposed actions are also consistent with equilibrium conditions, note that as before, H types have an incentive to choose their preferred strategies, knowing that (given voters' beliefs) they will be re-elected to implement them. Finally, note that in equilibrium, L types will prefer deviating from the proposed strategy if and only if exactly the same condition as in Claim 5 is satisfied (for pooling with the patient types):

$$u(X_1^H) + \delta^L V_{L,L}(X_1^H) \geq u(X_1^L) + pr\delta^L V_{L,H}(X_1^L) + (1-pr)\delta^L V_{L,L}(X_1^L)$$

[2]Note further that with a different belief for ranges between $X_t^*(\delta^H)$ and $X_t^*(\delta^L)$ low types may have an incentive to reduce expenditure from $X_t^*(\delta^L)$ but high types will never have an incentive to increase expenditure from $X_t^*(\delta^H)$, thus these beliefs are consistent even with out of equilibrium states in which policy makers expect beliefs other than these equilibrium beliefs.

and so they will prefer the proposed action if and only if the reverse condition holds, or equivalently if and only if:

$$p \leq \frac{1}{r} \frac{V_L^* - [u(X_1^H) + \delta^L V_{L,L}(X_1^H)]}{\delta^L [V_{L,L}(X_1^L) - V_{L,H}(X_1^L)]}$$

which is the condition in the claim. ■

These results demonstrate that if transparency is sufficiently high, electoral politics is sufficient to restrain impatient public officials and induce them to spend in moderation in early periods. If it is low, impatient incumbents ignore voters and implement their own preferred policies in Period 1. Note that the effectiveness of transparency depends on the options available to voters. If r is low, so that most policy makers are impatient, then voters cannot use the threat of removal from office to induce impatient policy makers to moderate their first-period spending. If however r is high and hence there exists a large pool of patient replacements, then the voters' threat has bite.

Note as well that for the threat of removal from office to be effective, impatient policy makers must not be *too* impatient. With sufficiently low values of δ, impatient policy makers will not respond to risks of future removals from office.